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Partially mode-dependent design of H_∞ filter for stochastic Markovian jump systems with mode-dependent time delays ☆

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ABSTRACT

This paper is concerned with the H_∞ filtering problem for stochastic delay systems with Markovian jump parameters, where both the state dynamics and measurements of systems are corrupted by Wiener process. In contrast with traditional mode-dependent and mode-independent filtering methods, a new partially mode-dependent filter is established via using a mode-dependent Lyapunov function, where the system mode available to filter implementation is transmitted through an unreliable network and the stochastic property of mode available to a filter is considered. Sufficient conditions for the existence of H_∞ filters are obtained as linear matrix inequalities. Finally, an example is used to show the effectiveness of the given theoretical results.

1. Introduction

Filtering has been one of the key problems in the areas of control and signal processing, which is used to estimate the unavailable state variables of a given system through noisy measurements. It is well known that the traditional Kalman filtering is an effective way to deal with the state estimation problems [1,2], which requires the exact knowledge of system model and the statics of external noise signal. When a priori information on external noise is not precisely known or system drifts, the celebrated Kalman filtering scheme is no longer applicable and has poor robustness performance. These features have motivated the study of H_∞ filtering problem for variant systems, see [3–6], and the references therein.

On the other hand, time delay is very common in practical dynamical systems, such as in chemical systems, manufacturing systems, biological systems, networked control systems (NCSs), telecommunication and economic systems, etc. It has been shown that the presence of time delay in a dynamical system is often a big source of instability and poor performance, and various research topics on delay system have been reported in recent years. In addition, Markovian jump systems (MJSs) can be regarded as a special class of hybrid systems with finite operation modes whose structures are subject to random abrupt changes [7]. The studies of MJSs are important in practical applications such as manufacturing systems, aircraft control, target tracking, robotics, solar receiver control, and power systems. Therefore, a great deal of attention has been devoted to the study of this class of systems in recent years such as [8–15], where the H_∞ filtering problem for MJSs were discussed in [16–18]. For the case of MJSs with time delays, the filtering results can be found in [19–21]. Especially, there is a special time delay for MJSs, which is named as mode-dependent time delay. Different from the aforesaid references, where the time delays are independent of system modes, another kind of time delay referred to be mode-dependent for MJS has been studied recently. The character of mode-dependent time delay is not only time-varying but also dependent on the system

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mode. It means that for any given system mode, the corresponding system becomes one with time-varying delay, whose time delay is more general in terms of theory and application. In this sense, it is said that MJSs with mode-dependent delays are worthwhile and significant to investigate. For MJSs with mode-dependent time-varying delays, the robust H_∞ filtering for uncertain MJSs was reported in [22], whose results were further extended in [23–25], and less conservative results referred to be delay-dependent were given in [26].

However, all of the aforementioned results on MJSs have an indispensable assumption that the current Markov mode is available to filter or controller implementation at every instant which are usually called to be mode-dependent. However, in many practical applications, the mode information is usually transmitted through unreliable networks, which suffers being lost and observed simultaneously instead of being obtained online. In this case, the aforementioned ideal assumption is impossible to satisfy, which limits the scope of applications of the established criteria. In order to deal with the aforesaid case that the system mode is inaccessible, [27] and [28] proposed mode-independent filtering methods, where the mode information is totally ignored no matter whether it is obtained or not. Since the system mode transmitted through networks suffers being lost and observed simultaneously, mode-dependent method is too ideal, while mode-independent algorithm is too absolute. As a result, both of the two extreme filter design method are not suitable to the case where the system mode is available to filter with some distribution property, and new filtering method should be established. To the best of our knowledge, the problem of H_∞ filtering for continuous-time MJSs with mode-dependent time-varying delays and system mode available to filter in terms of stochastic probability has not been fully investigated, which motivates the current research.

In this paper, the problem of H_∞ filtering for stochastic MJSs with mode-dependent time-varying delays is revisited, where a novel H_∞ filtering referred to be partially mode-dependent is proposed. Compared with traditional methods of MJSs, the desired filter is obtained by exploiting mode-dependent Lyapunov function instead of mode-independent (common) Lyapunov function, where the accessible probability of mode is taken into consideration for the filter design. In addition, different from the existing results for continuous-time MJSs with mode-dependent time-varying delays, sufficient conditions for the existence of partially mode-dependent H_∞ filer are established, where the matrices related to Lyapunov function are mode-dependent and are not assumed to be diagonal matrices. Finally, the advantages of the proposed method are also illustrated by an example.

Notation. $\mathbb{R}^n$ denotes the n-dimensional Euclidean space, $\mathbb{R}^{m\times n}$ is the set of all $m\times n$ real matrices. $\sigma_{\max}(M)$ and $\sigma_{\min}(M)$ denote the maximum and minimum singular value of matrix M respectively. $\mathscr{E}(\cdot)$ is the expectation operator with respect to some probability measure. In symmetric block matrices, we use "$*$" as an ellipsis for the terms induced by symmetry, $\operatorname{diag}\{\cdots\}$ for a block-diagonal matrix, and $(M)^\star \triangleq M + M^T$.

2. Problem formulation

Consider a class of stochastic time delay Markovian jump systems described as

$$\begin{cases} dx(t) = \big(A(\eta_t)x(t) + A_d(\eta_t)x\big(t-\tau_{\eta_t}(t)\big) + B(\eta_t)\upsilon(t)\big)\,dt + \big(G(\eta_t)x(t) + G_d(\eta_t)x\big(t-\tau_{\eta_t}(t)\big)\big)\,d\omega(t) \\ dy(t) = \big(C(\eta_t)x(t) + C_d(\eta_t)x\big(t-\tau_{\eta_t}(t)\big) + D(\eta_t)\upsilon(t)\big)\,dt + \big(H(\eta_t)x(t) + H_d(\eta_t)x\big(t-\tau_{\eta_t}(t)\big)\big)\,d\omega(t) \\ z(t) = L(\eta_t)x(t) \\ x(t) = \phi(t), \quad \forall t \in [-\overline{\mu}, 0] \end{cases} \tag{1}$$

where $x(t)\in\mathbb{R}^n$ is the state, $\upsilon(t)\in\mathbb{R}^p$ is the disturbance input which belongs to $\mathcal{L}_2[0,\infty)$, $y(t)\in\mathbb{R}^q$ is the measurement, $z(t)\in\mathbb{R}^m$ is the signal to be estimated. $\omega(t)$ is a one-dimensional standard Wiener process, which is independent of the Markov process. The parameter η_t is a continuous-time Markov process with right continuous trajectory taking values in a finite set $\mathbb{S}=\{1,2,\ldots,N\}$ with transition probability

$$\Pr\{\eta_{t+\delta}=j \mid \eta_t=i\} = \begin{cases} \lambda_{ij}\delta + o(\delta) & i\neq j \\ 1+\lambda_{ii}\delta + o(\delta) & i=j \end{cases} \tag{2}$$

where $\delta>0$, $\lim_{\delta\to 0^+}(o(\delta)/\delta)=0$ and the transition probability rate satisfies $\lambda_{ij}\geqslant 0$, for $i,j\in\mathbb{S}$, $i\neq j$ and

$$\lambda_{ii} = -\sum_{j=1,\,j\neq i}^{N} \lambda_{ij} \tag{3}$$

In system (1), $\tau_{\eta_t}(t)$ denotes time-varying delay, when the mode is in η_t and satisfies

$$0 \leqslant \underline{\mu}_i \leqslant \tau_i(t) \leqslant \overline{\mu}_i, \qquad \dot{\tau}_i(t) \leqslant h_i < 1, \quad \forall \eta_t = i \in \mathbb{S} \tag{4}$$

where $\overline{\mu}=\max\{\overline{\mu}_i,\ i\in\mathbb{S}\}$ and $\underline{\mu}=\min\{\underline{\mu}_i,\ i\in\mathbb{S}\}$. For notation simplicity, in the sequel, for each possible $\eta_t=i\in\mathbb{S}$, matrix $A(\eta_t)$ will be denoted by A_i, and so on.

In this paper, a novel filter called to be partially mode-dependent filter without time delay is described by

$$\begin{cases} dx_f(t) = A_f x_f(t)\,dt + B_f\,dy(t) + \alpha(t)\big(A_f(\eta_t)x_f(t)\,dt + B_f(\eta_t)\,dy(t)\big) \\ z_f(t) = \big(L_f + \alpha(t)L_f(\eta_t)\big)x_f(t) \end{cases} \tag{5}$$

where A_f, B_f, L_f, $A_f(\eta_t)$, $B_f(\eta_t)$ and $L_f(\eta_t)$ are filter parameters with appropriate dimensions to be determined. $\alpha(t)$ is an indicator function satisfying Bernoulli process and is described as

$$\alpha(t) = \begin{cases} 1 & \text{if } \eta_t \text{ is transmitted successfully} \\ 0 & \text{otherwise} \end{cases} \tag{6}$$

Then, we have

$$\Pr\{\alpha(t) = 1\} = \mathscr{E}\big(\alpha(t)\big) = \alpha, \qquad \Pr\{\alpha(t) = 0\} = 1 - \alpha \tag{7}$$

Moreover, it can be readily verified that

$$\mathscr{E}\big(\alpha(t) - \alpha\big) = 0, \qquad \beta^2 \triangleq \Pr\big\{\big(\alpha(t) - \alpha\big)^2\big\} = \alpha(1 - \alpha) \tag{8}$$

Remark 1. It is worth mentioning that results on MJSs can be generally classified into two categories: mode-dependent and mode-independent ones. Since mode-dependent criteria have a critical assumption that system mode needs to be obtained exactly, this ideal requirement sometimes inevitably limits the application of the proposed results. On the other hand, mode-independent method are usually applied to the case where the system mode is inaccessible. However, this method completely ignores the mode information even if the system mode is obtained sometimes, which is too absolute.

Remark 2. It is noted that, in many practical applications such as NCSs, data information is transmitted through networks, which is obtained or lost simultaneously. Both of the aforementioned methods are not suitable to this case. In this paper, a new partially mode-dependent filter of form (5) is presented, where the probability property of mode accessible to filter implementation is considered via a Bernoulli variable $\alpha(t)$. Compared with the traditional filtering methods, filter (5) is more advantageous in terms of the following two aspects. In contrast to mode-dependent filter with $\alpha(t) \equiv 1$, filter (5) can suffer the mode lost or drop the mode signal forwardly with some probability, which could reduce the burden of data transmission. Different from totally mode-independent filter with $\alpha(t) \equiv 0$, when the mode is accessible with some probability and there is no solution to mode-independent filter, we may still get an effective filter of form (5) or smaller minimum H_∞ performance γ^*. That is because mode-independent filter design method is to find a common filter for all modes, the solvable solution set is smaller than one generated by (5). In this sense, it is said that mode-independent filter design method is overdesign and is more conservative.

For any $\eta_t = i \in \mathbb{S}$, system (5) is equivalent to

$$\begin{cases} dx_f(t) = \big[\tilde{A}_{fi} + \big(\alpha(t) - \alpha\big)A_{fi}\big]x_f(t)\,dt + \big[\tilde{B}_{fi} + \big(\alpha(t) - \alpha\big)B_{fi}\big]\,dy(t) \\ z_f(t) = \big[\tilde{L}_{fi} + \big(\alpha(t) - \alpha\big)L_{fi}\big]x_f(t) \end{cases} \tag{9}$$

where

$$\tilde{A}_{fi} = A_f + \alpha A_{fi}, \qquad \tilde{B}_{fi} = B_f + \alpha B_{fi}, \qquad \tilde{L}_{fi} = L_f + \alpha L_{fi}$$

Connecting filter (5) to system (1), when $\eta_t = i \in \mathbb{S}$, we have the following filtering error system

$$\begin{cases} d\hat{x}(t) = \big(\hat{A}_i\hat{x}(t) + \hat{A}_{di}K\hat{x}\big(t - \tau_i(t)\big) + \hat{B}_i\upsilon(t)\big)\,dt + \big(\hat{G}_iK\hat{x}(t) + \hat{G}_{di}K\hat{x}\big(t - \tau_i(t)\big)\big)\,d\omega(t) \\ \qquad\quad + (\alpha(t) - \alpha)\big[\big(\check{A}_i\hat{x}(t) + \check{A}_{di}K\hat{x}\big(t - \tau_i(t)\big) + \check{B}_i\upsilon(t)\big)\,dt + \big(\check{G}_iK\hat{x}(t) + \check{G}_{di}K\hat{x}\big(t - \tau_i(t)\big)\big)\,d\omega(t)\big] \\ e(t) = \hat{L}_i\hat{x}(t) + \big(\alpha(t) - \alpha\big)\check{L}_i\hat{x}(t) \end{cases} \tag{10}$$

where

$$\hat{x}(t) = \begin{bmatrix} x(t) \\ x(t) - x_f(t) \end{bmatrix}, \qquad \hat{A}_i = \begin{bmatrix} A_i & 0 \\ A_i - \tilde{A}_{fi} - \tilde{B}_{fi}C_i & \tilde{A}_{fi} \end{bmatrix}, \qquad \hat{A}_{di} = \begin{bmatrix} A_{di} \\ A_{di} - \tilde{B}_{fi}C_{di} \end{bmatrix}$$

$$\hat{B}_i = \begin{bmatrix} B_i \\ B_i - \tilde{B}_{fi}D_i \end{bmatrix}, \qquad \hat{G}_i = \begin{bmatrix} G_i \\ G_i - \tilde{B}_{fi}H_i \end{bmatrix}, \qquad \hat{G}_{di} = \begin{bmatrix} G_{di} \\ G_{di} - \tilde{B}_{fi}H_{di} \end{bmatrix}$$

$$\check{A}_i = \begin{bmatrix} 0 & 0 \\ -B_{fi}C_i + A_{fi} & -A_{fi} \end{bmatrix}, \qquad \check{A}_{di} = \begin{bmatrix} 0 \\ -B_{fi}C_{di} \end{bmatrix}, \qquad \check{B}_i = \begin{bmatrix} 0 \\ -B_{fi}D_i \end{bmatrix}$$

$$\check{G}_i = \begin{bmatrix} 0 \\ -B_{fi}H_i \end{bmatrix}, \qquad \check{G}_{di} = \begin{bmatrix} 0 \\ -B_{fi}H_{di} \end{bmatrix}$$

$$e(t) = z(t) - z_f(t), \qquad K = \begin{bmatrix} I & 0 \end{bmatrix}, \qquad \hat{L}_i = \begin{bmatrix} L_i - \tilde{L}_{fi} & \tilde{L}_{fi} \end{bmatrix}, \qquad \check{L}_i = \begin{bmatrix} -L_{fi} & L_{fi} \end{bmatrix}$$

Before presenting the main results, some definitions are needed for filtering error system (10).

Definition 1. System (10) with $\upsilon(t) \equiv 0$ is said to be exponentially mean-square stable, if there exist constants $a > 0$ and $b > 0$ such that

$$\mathscr{E}\left(\left\|\hat{x}(t)\right\|^2 | \hat{x}_0, \eta_0\right) \leqslant ae^{-bt}\|\hat{x}_0\|^2 \tag{11}$$

for any initial conditions $\hat{x}_0 \in \mathbb{R}^n$ and $\eta_0 \in \mathbb{S}$.

Definition 2. Given a scalar $\gamma > 0$, system (10) is said to be exponentially mean-square stable with H_∞ performance, if it is exponentially mean-square stable and the filtering error $e(t)$ under zero initial condition and any nonzero $\upsilon(t) \in \mathcal{L}_2[0,\infty)$, satisfies

$$\mathscr{E}\left(\int_0^\infty e^T(t)e(t)\,dt\right) < \gamma^2 \int_0^\infty \upsilon^T(t)\upsilon(t)\,dt \tag{12}$$

3. H_∞ performance analysis

In this section, the H_∞ performance of filtering error system (10) will be given in terms of LMIs.

Theorem 1. *Given a scalar $\gamma > 0$, system* (10) *is exponentially mean-square stable with H_∞ performance, if there exist matrices $P_i > 0$ and $Q > 0$, such that the following LMIs hold for all $i \in \mathbb{S}$*

$$\begin{bmatrix} \Omega_i & P_i\hat{A}_{di} & P_i\hat{B}_i & K^T\hat{G}_i^T P_i & \beta K^T \check{G}_i^T P_i \\ * & -\hat{Q}_i & 0 & \hat{G}_{di}^T P_i & \beta \check{G}_{di}^T P_i \\ * & * & -\gamma^2 I & 0 & 0 \\ * & * & * & -P_i & 0 \\ * & * & * & * & -P_i \end{bmatrix} < 0 \tag{13}$$

where

$$\Omega_i = (P_i\hat{A}_i)^\star + (1 + \lambda\tilde{\mu})K^T Q K + \bar{P}_i + \hat{L}_i^T\hat{L}_i + \beta^2\check{L}_i^T\check{L}_i$$

$$\tilde{\mu} = (\overline{\mu} - \underline{\mu}), \qquad \bar{P}_i = \sum_{j=1}^N \lambda_{ij}P_j, \qquad \hat{Q}_i = (1 - h_i)Q, \qquad \lambda = \max\{|\lambda_{ii}|,\ i \in \mathbb{S}\}$$

Proof. Define a new process $\{(\hat{x}_t, \eta_t),\ t \geqslant 0\}$ with $\hat{x}_t(s) = \hat{x}(t+s)$, $t - \tau_{\eta_t}(t) \leqslant t+s \leqslant t$. From [29], we know $\{(\hat{x}_t, \eta(t)),\ t \geqslant 0\}$ is a Markov process. Now, choose a stochastic Lyapunov function for system (10) as

$$V(\hat{x}_t, \eta_t) = V_1(\hat{x}_t, \eta_t) + V_2(\hat{x}_t, \eta_t) + V_3(\hat{x}_t, \eta_t) \tag{14}$$

where

$$V_1(\hat{x}_t, \eta_t) = \hat{x}^T(t)P(\eta_t)\hat{x}(t)$$

$$V_2(\hat{x}_t, \eta_t) = \int_{t-\tau_{\eta_t}(t)}^{t} \hat{x}^T(s)K^T Q K\hat{x}(s)\,ds$$

$$V_3(\hat{x}_t, \eta_t) = \lambda \int_{-\overline{\mu}}^{-\underline{\mu}} \int_{t+\theta}^{t} \hat{x}^T(s)K^T Q K\hat{x}(s)\,ds\,d\theta$$

Let $\mathscr{A}$ be the weak infinitesimal generator of random process $\{\hat{x}_t, \eta_t\}$, for each $\eta(t) = i \in \mathbb{S}$, it is defined as

$$\mathscr{A}\left[V(\hat{x}_t, \eta_t = i)\right] = \lim_{\delta \to 0^+} \frac{1}{\delta}\left[\mathscr{E}\left(V\left(\hat{x}(t+\delta), \eta_{t+\delta}\right) | \hat{x}(t), \eta_t = i\right) - V\left(\hat{x}(t), i\right)\right] \tag{15}$$

Then under $\upsilon(t) \equiv 0$, we have

$$\begin{aligned}\mathscr{A}\big[V_1(\hat{x}_t, \eta_t)\big] &= 2\hat{x}^T(t)P_i\big(A_i\hat{x}(t) + A_{di}K\hat{x}\big(t-\tau_i(t)\big)\big) + \hat{x}^T(t)\bar{P}_i\hat{x}(t) \\ &\quad + \big(\hat{G}_iK\hat{x}(t) + \hat{G}_{di}K\hat{x}\big(t-\tau_i(t)\big)\big)^T P_i\big(\hat{G}_iK\hat{x}(t) + \hat{G}_{di}K\hat{x}\big(t-\tau_i(t)\big)\big) \\ &\quad + \beta^2\big(\check{G}_iK\hat{x}(t) + \check{G}_{di}K\hat{x}\big(t-\tau_i(t)\big)\big)^T P_i\big(\check{G}_iK\hat{x}(t) + \check{G}_{di}K\hat{x}\big(t-\tau_i(t)\big)\big)\end{aligned} \tag{16}$$

Similarly, we have

$$\mathscr{A}\big[V_2(\hat{x}_t, \eta_t)\big] \leqslant \hat{x}^T(t)K^TQK\hat{x}(t) - (1-h_i)\hat{x}^T\big(t-\tau_i(t)\big)K^TQK\hat{x}\big(t-\tau_i(t)\big) + \sum_{j=1}^{N}\lambda_{ij}\int_{t-\tau_j(t)}^{t}\hat{x}^T(s)K^TQK\hat{x}(s)\,ds \tag{17}$$

$$\mathscr{A}\big[V_3(\hat{x}_t, \eta_t)\big] = \lambda\tilde{\mu}\hat{x}^T(t)K^TQK\hat{x}(t) - \lambda\int_{t-\overline{\mu}}^{t-\underline{\mu}}\hat{x}^T(s)K^TQK\hat{x}(s)\,ds \tag{18}$$

Moreover, we have

$$\begin{aligned}\sum_{j=1}^{N}\lambda_{ij}\int_{t-\tau_j(t)}^{t}\divideontimes\,ds &= \sum_{j\neq i}\lambda_{ij}\int_{t-\tau_j(t)}^{t}\divideontimes\,ds + \lambda_{ii}\int_{t-\tau_i(t)}^{t}\divideontimes\,ds \leqslant \sum_{j\neq i}\lambda_{ij}\int_{t-\overline{\mu}}^{t}\divideontimes\,ds + \lambda_{ii}\int_{t-\underline{\mu}}^{t}\divideontimes\,ds \\ &= -\lambda_{ii}\int_{t-\overline{\mu}}^{t-\underline{\mu}}\divideontimes\,ds \leqslant \lambda\int_{t-\overline{\mu}}^{t-\underline{\mu}}\divideontimes\,ds\end{aligned} \tag{19}$$

where $\divideontimes \triangleq \hat{x}^T(s)K^TQK\hat{x}(s)$.

Taking into account (16)–(19), we get

$$\mathscr{A}\big[V(\hat{x}_t, \eta_t)\big] \leqslant \xi^T(t)\Upsilon(\eta_t)\xi(t) \tag{20}$$

where

$$\xi^T(t) = \begin{bmatrix}\hat{x}^T(t) & \hat{x}^T(t-\tau_i(t))K^T\end{bmatrix}$$

$$\Upsilon_i = \begin{bmatrix}(P_i\hat{A}_i)^\star + (1+\lambda\tilde{\mu})K^TQK + \bar{P}_i & P_i\hat{A}_{di} \\ * & -\hat{Q}_i\end{bmatrix} + \begin{bmatrix}\hat{G}_i^TK^T \\ \hat{G}_{di}^T\end{bmatrix}P_i\begin{bmatrix}\hat{G}_i^TK^T \\ \hat{G}_{di}^T\end{bmatrix}^T + \beta^2\begin{bmatrix}\check{G}_i^TK^T \\ \check{G}_{di}^T\end{bmatrix}P_i\begin{bmatrix}\check{G}_i^TK^T \\ \check{G}_{di}^T\end{bmatrix}^T$$

From (13), we obtain $\Upsilon_i < 0$, then there always exists a sufficiently small scalar $\varepsilon > 0$ such that

$$\mathscr{A}\big[V(\hat{x}_t, \eta_t)\big] \leqslant -\varepsilon\hat{x}^T(t)\hat{x}(t) \tag{21}$$

By Dynkin's formula, we have for each $\eta_T = i \in \mathbb{S}$ and $T > 0$

$$\mathscr{E}\big(V(\hat{x}_T, \eta_T)\big) - \mathscr{E}\big(V(\hat{x}_0, \eta_0)\big) = \mathscr{E}\left\{\int_0^T\mathscr{A}\big[V(\hat{x}_T, \eta_T)\big]\right\} \leqslant -\varepsilon\int_0^T\mathscr{E}\big(\hat{x}^T(s)\hat{x}(s)\big)\,ds \tag{22}$$

On the other hand, it follows from (14) that

$$\mathscr{E}\big(V(\hat{x}_t, \eta_t)\big) \geqslant a\mathscr{E}\big(\hat{x}^T(t)\hat{x}(t)\big) \tag{23}$$

where $a = \min_{i\in\mathbb{S}}\{\lambda_{\min}(P_i)\} > 0$. From (22) and (23), we have

$$\mathscr{E}\big(\hat{x}^T(t)\hat{x}(t)\big) \leqslant a^{-1}V(\hat{x}_0, \eta_0) - a^{-1}\varepsilon\int_0^T\mathscr{E}\big(\hat{x}^T(s)\hat{x}(s)\big)\,ds \tag{24}$$

Then applying the Gronwall–Bellman lemma to (24), we can get

$$\mathscr{E}\big(\hat{x}^T(T)\hat{x}(T)\big) \leqslant a^{-1}V(\hat{x}_0, \eta_0)\exp\big(-\varepsilon a^{-1}T\big) \tag{25}$$

It is concluded that there exists a scalar $b > 0$ such that

$$a^{-1} V(\hat{x}_0, \eta_0) \leqslant b \sup_{-\bar{\mu} \leqslant s \leqslant 0} \|\hat{x}(s)\|^2 \tag{26}$$

then we can get (10) is exponentially mean-square stable.

Now, we will show the H_∞ performance of system (10) with zero initial condition and nonzero $\upsilon(t) \in \mathcal{L}_2[0, \infty)$. Define

$$J_T \triangleq \mathscr{E}\left\{ \int_0^T \left(e^T(t)e(t) - \gamma^2 \upsilon^T(t)\upsilon(t)\right) dt \right\} \tag{27}$$

Then, we have

$$\begin{aligned} J_T &= \mathscr{E}\left\{ \int_0^T \left(e^T(t)e(t) - \gamma^2 \upsilon^T(t)\upsilon(t) + \mathscr{A}\left[V\left(\hat{x}(t), \eta_t\right)\right]\right) dt \right\} - \mathscr{E}\left\{V\left(\hat{x}(T), \eta_T\right)\right\} \\ &\leqslant \mathscr{E}\left(\int_0^T \hat{\xi}^T(t)\hat{\Gamma}(\eta_t)\hat{\xi}(t)\, dt \right) < 0 \end{aligned} \tag{28}$$

where

$$\hat{\xi}^T(t) = \begin{bmatrix} \xi^T(t) & \upsilon^T(t) \end{bmatrix}$$

$$\hat{\Gamma}_i = \begin{bmatrix} \Omega_i & P_i\hat{A}_{di} & P_i\hat{B}_i \\ * & -\hat{Q}_i & 0 \\ * & * & -\gamma^2 I \end{bmatrix} + \begin{bmatrix} \hat{G}_i^T K^T \\ \hat{G}_{di}^T \\ 0 \end{bmatrix} P_i \begin{bmatrix} \hat{G}_i^T K^T \\ \hat{G}_{di}^T \\ 0 \end{bmatrix}^T + \beta^2 \begin{bmatrix} \check{G}_i^T K^T \\ \check{G}_{di}^T \\ 0 \end{bmatrix} P_i \begin{bmatrix} \check{G}_i^T K^T \\ \check{G}_{di}^T \\ 0 \end{bmatrix}^T$$

Since (14) is equivalent to $\hat{\Gamma}_i < 0$, it implies (29) holds. Then, we have (12) holds. This completes the proof. □

Remark 3. It is worth remarked that the Lyapunov function in Theorem 1 is similar to ones in [22,24,25], which could also be chosen as other forms to further reduce the conservatism such as in [26]. However, there is a contradiction between exploiting mode-dependent Lyapunov function and designing partially mode-dependent filter (5) simultaneously, where the filter parameters are not easy to be solved.

Theorem 2. *Given a scalar $\gamma > 0$, system (10) is exponentially mean-square stable with H_∞ performance, if there exist matrices $P_i > 0$, $Q > 0$, X_i and Y_i, such that the following LMIs hold for all $i \in \mathbb{S}$*

$$\begin{bmatrix} \Sigma_{i11} & \Sigma_{i12} \\ * & \Sigma_{i22} \end{bmatrix} < 0 \tag{29}$$

where

$$\Sigma_{i11} = \begin{bmatrix} -(\frac{X_i}{2})^\star & \Sigma_{i11}^2 & 0 & 0 \\ * & -(Y_i)^\star & 0 & 0 \\ * & * & -(\frac{X_i}{2})^\star & \Sigma_{i11}^2 \\ * & * & * & -(Y_i)^\star \end{bmatrix}, \qquad \Sigma_{i12} = \begin{bmatrix} \beta X_i^T \check{G}_i K & 0 & \beta X_i^T \check{G}_{di} & 0 \\ \beta Y_i^T \check{G}_i K & 0 & \beta Y_i^T \check{G}_{di} & 0 \\ X_i^T \hat{G}_i K & 0 & X_i^T \hat{G}_{di} & 0 \\ Y_i^T \hat{G}_i K & 0 & Y_i^T \hat{G}_{di} & 0 \end{bmatrix}$$

$$\Sigma_{i22} = \begin{bmatrix} \Sigma_{i22}^1 & \Sigma_{i22}^2 & X_i^T \hat{A}_{di} & X_i^T \hat{B}_i \\ * & -(Y_i)^\star & Y_i^T \hat{A}_{di} & Y_i^T \hat{B}_{di} \\ * & * & -\hat{Q}_i & 0 \\ * & * & * & -\gamma^2 I \end{bmatrix}$$

$$\Sigma_{i11}^2 = -\frac{Y_i}{2} + P_i - X_i^T$$

$$\Sigma_{i22}^1 = \left(\hat{A}_i^T X_i\right)^\star + (1 + \lambda\tilde{\mu})K^T Q K + \bar{P}_i + \hat{L}_i^T \hat{L}_i + \beta^2 \check{L}_i^T \check{L}_i, \qquad \Sigma_{i22}^2 = \hat{A}_i^T Y_i + P_i - X_i^T$$

Proof. From Theorem 1, it is readily verified that condition (13) is equivalent to

$$\begin{bmatrix} -P_i & 0 & \beta P_i \check{G}_i K & \beta P_i \check{G}_{di} & 0 \\ * & -P_i & P_i \hat{G}_i & P_i \hat{G}_{di} & 0 \\ * & * & \Omega_i & P_i \hat{A}_{di} & P_i \hat{B}_i \\ * & * & * & -\hat{Q}_i & 0 \\ * & * & * & * & -\gamma^2 I \end{bmatrix} < 0 \tag{30}$$

which is obtained by pre- and post-multiplying with the following matrix

$$\begin{bmatrix} 0 & 0 & 0 & 0 & I \\ 0 & 0 & 0 & I & 0 \\ I & 0 & 0 & 0 & 0 \\ 0 & I & 0 & 0 & 0 \\ 0 & 0 & I & 0 & 0 \end{bmatrix}$$

and its transpose, respectively. If condition (30) holds, system (10) is exponentially mean-square stable with H_∞ performance. Now, pre- and post-multiplying (29) with the following matrix

$$\begin{bmatrix} I & -\frac{I}{2} & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & I & -\frac{I}{2} & 0 & 0 & 0 & 0 \\ 0 & \beta K^T \check{G}_i^T & 0 & K^T \hat{G}_i^T & I & \hat{A}_i^T & 0 & 0 \\ 0 & \beta \check{G}_{di}^T & 0 & \hat{G}_{di}^T & 0 & \hat{A}_{di}^T & I & 0 \\ 0 & 0 & 0 & 0 & 0 & \hat{B}_i^T & 0 & I \end{bmatrix}$$

and its transpose, respectively. Then, it is directly obtained that (29) implies (30). This completes the proof. □

Remark 4. It is seen that Theorem 2 proposes another sufficient condition for exponentially mean-square stable with H_∞ performance of error system (10). More important, it separates Lyapunov matrix P_i from system matrices such as $\hat{A}_i$, $\hat{B}_i$, and so on, which makes the requirements of partially mode-dependent filter and mode-dependent Lyapunov function are likely to be satisfied simultaneously. In addition, similar criteria of stochastic MJSs with time-varying parameter uncertainties can be established.

4. H_∞ filter design

In this section, we will give LMI conditions for the existence of partially mode-dependent filter of form (5).

Theorem 3. *Given a scalar $\gamma > 0$, system* (10) *is exponentially mean-square stable with H_∞ performance, if there exist matrices $P_{i1} > 0$, P_{i2}, $P_{i3} > 0$, $Q > 0$, X_{i1}, X_{i2}, Y_{i1}, Y_{i2}, Z, $\bar{A}_f$, $\bar{B}_f$, $\bar{L}_f$, $\bar{A}_{fi}$, $\bar{B}_{fi}$ and $\bar{L}_{fi}$, such that the following LMIs hold for all $i \in \mathbb{S}$*

$$\begin{bmatrix} \hat{\Sigma}_{i11} & \hat{\Sigma}_{i12} & 0 \\ & \hat{\Sigma}_{i22} & \hat{\Sigma}_{i23} \\ & * & \hat{\Sigma}_{i33} \end{bmatrix} < 0 \tag{31}$$

$$\begin{bmatrix} P_{i1} & P_{i2} \\ & P_{i3} \end{bmatrix} > 0 \tag{32}$$

where

$$\hat{\Sigma}_{i11} = \begin{bmatrix} \hat{\Sigma}_{i11}^1 & \hat{\Sigma}_{i11}^2 & 0 & 0 \\ * & \hat{\Sigma}_{i11}^3 & 0 & 0 \\ * & * & \hat{\Sigma}_{i11}^1 & \hat{\Sigma}_{i11}^2 \\ * & * & * & \hat{\Sigma}_{i11}^3 \end{bmatrix}, \qquad \hat{\Sigma}_{i12} = \begin{bmatrix} \hat{\Sigma}_{i12}^1 & 0 & \hat{\Sigma}_{i12}^2 & 0 \\ \hat{\Sigma}_{i12}^1 & 0 & \hat{\Sigma}_{i12}^2 & 0 \\ \hat{\Sigma}_{i12}^3 & 0 & \hat{\Sigma}_{i12}^4 & 0 \\ \hat{\Sigma}_{i12}^5 & 0 & \hat{\Sigma}_{i12}^6 & 0 \end{bmatrix}$$

$$\hat{\Sigma}_{i22} = \begin{bmatrix} \hat{\Sigma}_{i22}^1 & \hat{\Sigma}_{i22}^2 & \hat{\Sigma}_{i22}^3 & \hat{\Sigma}_{i22}^4 \\ * & \hat{\Sigma}_{i11}^3 & \hat{\Sigma}_{i22}^5 & \hat{\Sigma}_{i22}^6 \\ * & * & -\hat{Q}_i & 0 \\ * & * & * & -\gamma^2 I \end{bmatrix}, \qquad \hat{\Sigma}_{i23} = \begin{bmatrix} \hat{\Sigma}_{i23}^1 & \hat{\Sigma}_{i23}^2 \\ 0 & 0 \\ 0 & 0 \\ 0 & 0 \end{bmatrix}, \qquad \hat{\Sigma}_{i33} = \begin{bmatrix} -I & 0 \\ * & -I \end{bmatrix}$$

$$\hat{\Sigma}^1_{i11} = -\frac{1}{2}\begin{bmatrix} (X_{i1})^\star & X^T_{i2} + Z \\ * & (Z)^\star \end{bmatrix}, \qquad \hat{\Sigma}^2_{i11} = \begin{bmatrix} -\frac{1}{2}Y^T_{i1} + P_{i1} - X_{i1} & -\frac{1}{2}Y^T_{i2} + P_{i2} - Z \\ -\frac{1}{2}Z^T + P^T_{i2} - X_{i2} & -\frac{1}{2}Z^T + P_{i3} - Z \end{bmatrix}$$

$$\hat{\Sigma}^3_{i11} = -\begin{bmatrix} (Y_{i1})^\star & Y^T_{i2} + Z \\ * & (Z)^\star \end{bmatrix}, \qquad \hat{\Sigma}^1_{i12} = \beta\begin{bmatrix} -\bar{B}_{fi}H_i & 0 \\ -\bar{B}_{fi}H_i & 0 \end{bmatrix}, \qquad \hat{\Sigma}^2_{i12} = \beta\begin{bmatrix} -\bar{B}_{fi}H_{di} \\ -\bar{B}_{fi}H_{di} \end{bmatrix}$$

$$\hat{\Sigma}^3_{i12} = \begin{bmatrix} X_{i1}G_i + ZG_i - \breve{B}_{fi}H_i & 0 \\ X_{i2}G_i + ZG_i - \breve{B}_{fi}H_i & 0 \end{bmatrix}, \qquad \hat{\Sigma}^4_{i12} = \begin{bmatrix} X_{i1}G_i + ZG_i - \breve{B}_{fi}H_{di} \\ X_{i2}G_i + ZG_i - \breve{B}_{fi}H_{di} \end{bmatrix}$$

$$\hat{\Sigma}^5_{i12} = \begin{bmatrix} Y_{i1}G_i + ZG_i - \breve{B}_{fi}H_i & 0 \\ Y_{i2}G_i + ZG_i - \breve{B}_{fi}H_i & 0 \end{bmatrix}, \qquad \hat{\Sigma}^6_{i12} = \begin{bmatrix} Y_{i1}G_i + ZG_i - \breve{B}_{fi}H_{di} \\ Y_{i2}G_i + ZG_i - \breve{B}_{fi}H_{di} \end{bmatrix}$$

$$\hat{\Sigma}^1_{i22} = \begin{bmatrix} \hat{\Sigma}^{11}_{i22} & \hat{\Sigma}^{12}_{i22} \\ * & \hat{\Sigma}^{13}_{i22} \end{bmatrix}, \qquad \hat{\Sigma}^2_{i22} = \begin{bmatrix} \hat{\Sigma}^{21}_{i22} & \hat{\Sigma}^{22}_{i22} \\ \hat{\Sigma}^{23}_{i22} & \hat{\Sigma}^{24}_{i22} \end{bmatrix}$$

$$\hat{\Sigma}^3_{i22} = \begin{bmatrix} X_{i1}A_{di} + ZA_{di} - \breve{B}_{fi}C_{di} \\ X_{i2}A_{di} + ZA_{di} - \breve{B}_{fi}C_{di} \end{bmatrix}, \qquad \hat{\Sigma}^4_{i22} = \begin{bmatrix} X_{i1}B_i + ZB_i - \breve{B}_{fi}D_i \\ X_{i2}B_i + ZB_i - \breve{B}_{fi}D_i \end{bmatrix}$$

$$\hat{\Sigma}^5_{i22} = \begin{bmatrix} Y_{i1}A_{di} + ZA_{di} - \breve{B}_{fi}C_{di} \\ Y_{i2}A_{di} + ZA_{di} - \breve{B}_{fi}C_{di} \end{bmatrix}, \qquad \hat{\Sigma}^6_{i22} = \begin{bmatrix} Y_{i1}B_i + ZB_i - \breve{B}_{fi}D_i \\ Y_{i2}B_i + ZB_i - \breve{B}_{fi}D_i \end{bmatrix}$$

$$\hat{\Sigma}^1_{i23} = \begin{bmatrix} L^T_i - \breve{L}^T_{fi} \\ \breve{L}^T_{fi} \end{bmatrix}, \qquad \hat{\Sigma}^2_{i23} = \beta\begin{bmatrix} -\breve{L}^T_{fi} \\ \breve{L}^T_{fi} \end{bmatrix}$$

$$\hat{\Sigma}^{11}_{i22} = (X_{i1}A_i + ZA_i - \breve{A}_{fi} - \breve{B}_{fi}C_i)^\star + \bar{P}_{i1} + (1 + \lambda\tilde{\mu})Q$$

$$\hat{\Sigma}^{12}_{i22} = \breve{A}_{fi} + A^T_i X^T_{i2} + A^T_i Z^T - \breve{A}^T_{fi} - C^T_i \breve{B}^T_{fi} + \bar{P}_{i2}, \qquad \hat{\Sigma}^{13}_{i22} = (\breve{A}_{fi})^\star + \bar{P}_{i3}$$

$$\hat{\Sigma}^{21}_{i22} = A^T_i Y^T_{i1} + A^T_i Z^T - \breve{A}^T_{fi} - C^T_i \breve{B}^T_{fi} + P_{i1} - X_{i1}$$

$$\hat{\Sigma}^{22}_{i22} = A^T_i Y^T_{i2} + A^T_i Z^T - \breve{A}^T_{fi} - C^T_i \breve{B}^T_{fi} + P_{i2} - Z, \qquad \hat{\Sigma}^{23}_{i22} = \breve{A}^T_{fi} + P^T_{i2} - X_{i2}$$

$$\hat{\Sigma}^{24}_{i22} = \breve{A}^T_{fi} + P_{i3} - Z, \qquad \bar{P}_{i1} = \sum_{j=1}^{N} \lambda_{ij} P_{j1}, \qquad \bar{P}_{i2} = \sum_{j=1}^{N} \lambda_{ij} P_{j2}, \qquad \bar{P}_{i3} = \sum_{j=1}^{N} \lambda_{ij} P_{j3}$$

$$\breve{A}_{fi} = \bar{A}_f + \alpha\bar{A}_{fi}, \qquad \breve{B}_{fi} = \bar{B}_f + \alpha\bar{B}_{fi}, \qquad \breve{L}_{fi} = \bar{L}_f + \alpha\bar{L}_{fi}$$

Then, the parameters of filter (5) *can be constructed by*

$$A_f = Z^{-1}\bar{A}_f, \qquad B_f = Z^{-1}\bar{B}_f, \qquad L_f = \bar{L}_f, \qquad A_{fi} = Z^{-1}\bar{A}_{fi}, \qquad B_{fi} = Z^{-1}\bar{B}_{fi}, \qquad L_{fi} = \bar{L}_{fi} \tag{33}$$

Proof. If matrices P_{i1}, P_{i2}, P_{i3}, Q, X_{i1}, X_{i2}, Y_{i1}, Y_{i2} and Z are solutions to Theorem 3, matrices X_i, Y_i and P_i are of the following forms, that is

$$P_i = \begin{bmatrix} P_{i1} & P_{i2} \\ & P_{i3} \end{bmatrix}, \qquad X^T_i = \begin{bmatrix} X_{i1} & Z \\ X_{i2} & Z \end{bmatrix}, \qquad Y^T_i = \begin{bmatrix} Y_{i1} & Z \\ Y_{i2} & Z \end{bmatrix} \tag{34}$$

Then, taking into account (33), it is readily seen that Theorem 2 holds. This completes the proof. □

Remark 5. Theorem 3 presents a sufficient strict LMI condition for designing a partially mode-dependent filter via using mode-dependent Lyapunov function, where the filtering process has a fast convergence and acceptable accuracy in terms of reasonable error covariance can be guaranteed. For MJSs with mode-dependent time-varying delays, the existing filtering methods are totally mode-dependent, which cannot be applied to the aforesaid problem. In order to get the filter parameter, matrix P_i of mode-dependent Lyapunov function in the aforementioned references is assumed to be diagonal matrix. However, by Theorem 3, we can solve them separately and synchronously, even if filter (5) becomes totally mode-dependent. Moreover, from Theorem 3, it is seen that the mode accessible probability α is involved, which plays an important role in partially mode-dependent filter design.

When the system mode is always unavailable to filter, in this case, we may design a totally mode-independent filter with $\alpha(t) \equiv 0$ in (5), that is

$$\begin{cases} dx_f(t) = A_f x_f(t)\,dt + B_f\,dy(t) \\ z_f(t) = L_f x_f(t) \end{cases} \tag{35}$$

where A_f, B_f and L_f are mode-independent filter parameters with appropriate dimensions to be determined. Then, via the similar method, we will get the following corollary.

Corollary 1. *Given a scalar $\gamma > 0$, system* (10) *is exponentially mean-square stable with H_∞ performance, if there exist matrices $P_{i1} > 0$, P_{i2}, $P_{i3} > 0$, $Q > 0$, X_{i1}, X_{i2}, Y_{i1}, Y_{i2}, Z, $\bar{A}_f$, $\bar{B}_f$ and $\bar{L}_f$, such that the following LMIs hold for all $i \in \mathbb{S}$*

$$\begin{bmatrix} \tilde{\Sigma}_{i11} & \tilde{\Sigma}_{i12} & 0 \\ * & \tilde{\Sigma}_{i22} & \tilde{\Sigma}_{i23} \\ * & * & -I \end{bmatrix} < 0 \tag{36}$$

$$\begin{bmatrix} P_{i1} & P_{i2} \\ * & P_{i3} \end{bmatrix} > 0 \tag{37}$$

where

$$\tilde{\Sigma}_{i11} = \begin{bmatrix} \hat{\Sigma}_{i11}^1 & \hat{\Sigma}_{i11}^2 \\ * & \hat{\Sigma}_{i11}^3 \end{bmatrix}, \qquad \tilde{\Sigma}_{i12} = \begin{bmatrix} \tilde{\Sigma}_{i12}^1 & 0 & \tilde{\Sigma}_{i12}^2 & 0 \\ \tilde{\Sigma}_{i12}^3 & 0 & \tilde{\Sigma}_{i12}^4 & 0 \end{bmatrix}$$

$$\tilde{\Sigma}_{i22} = \begin{bmatrix} \tilde{\Sigma}_{i22}^1 & \tilde{\Sigma}_{i22}^2 & \tilde{\Sigma}_{i22}^3 & \tilde{\Sigma}_{i22}^4 \\ * & \hat{\Sigma}_{i11}^3 & \tilde{\Sigma}_{i22}^5 & \tilde{\Sigma}_{i22}^6 \\ * & * & -\hat{Q}_i & 0 \\ * & * & * & -\gamma^2 I \end{bmatrix}, \qquad \tilde{\Sigma}_{i23} = \begin{bmatrix} \tilde{\Sigma}_{i23}^1 \\ 0 \\ 0 \\ 0 \end{bmatrix}$$

$$\tilde{\Sigma}_{i12}^1 = \begin{bmatrix} X_{i1}G_i + ZG_i - \bar{B}_f H_i & 0 \\ X_{i2}G_i + ZG_i - \bar{B}_f H_i & 0 \end{bmatrix}, \qquad \tilde{\Sigma}_{i12}^2 = \begin{bmatrix} X_{i1}G_i + ZG_i - \bar{B}_f H_{di} \\ X_{i2}G_i + ZG_i - \bar{B}_f H_{di} \end{bmatrix}$$

$$\tilde{\Sigma}_{i12}^3 = \begin{bmatrix} Y_{i1}G_i + ZG_i - \bar{B}_f H_i & 0 \\ Y_{i2}G_i + ZG_i - \bar{B}_f H_i & 0 \end{bmatrix}, \qquad \tilde{\Sigma}_{i12}^4 = \begin{bmatrix} Y_{i1}G_i + ZG_i - \bar{B}_f H_{di} \\ Y_{i2}G_i + ZG_i - \bar{B}_f H_{di} \end{bmatrix}$$

$$\tilde{\Sigma}_{i22}^1 = \begin{bmatrix} \tilde{\Sigma}_{i22}^{11} & \tilde{\Sigma}_{i22}^{12} \\ * & \tilde{\Sigma}_{i22}^{13} \end{bmatrix}, \qquad \tilde{\Sigma}_{i22}^2 = \begin{bmatrix} \tilde{\Sigma}_{i22}^{21} & \tilde{\Sigma}_{i22}^{22} \\ \tilde{\Sigma}_{i22}^{23} & \tilde{\Sigma}_{i22}^{24} \end{bmatrix}$$

$$\tilde{\Sigma}_{i22}^3 = \begin{bmatrix} X_{i1}A_{di} + ZA_{di} - \bar{B}_f C_{di} \\ X_{i2}A_{di} + ZA_{di} - \bar{B}_f C_{di} \end{bmatrix}, \qquad \tilde{\Sigma}_{i22}^4 = \begin{bmatrix} X_{i1}B_i + ZB_i - \bar{B}_f D_i \\ X_{i2}B_i + ZB_i - \bar{B}_f D_i \end{bmatrix}$$

$$\tilde{\Sigma}_{i22}^5 = \begin{bmatrix} Y_{i1}A_{di} + ZA_{di} - \bar{B}_f C_{di} \\ Y_{i2}A_{di} + ZA_{di} - \bar{B}_f C_{di} \end{bmatrix}, \qquad \tilde{\Sigma}_{i22}^6 = \begin{bmatrix} Y_{i1}B_i + ZB_i - \bar{B}_f D_i \\ Y_{i2}B_i + ZB_i - \bar{B}_f D_i \end{bmatrix}, \qquad \tilde{\Sigma}_{i23}^1 = \begin{bmatrix} L_i^T - \bar{L}_f^T \\ \bar{L}_f^T \end{bmatrix}$$

$$\tilde{\Sigma}_{i22}^{11} = (X_{i1}A_i + ZA_i - \bar{A}_f - \bar{B}_f C_i)^\star + \bar{P}_{i1} + (1 + \lambda\tilde{\mu})Q$$

$$\tilde{\Sigma}_{i22}^{12} = \bar{A}_f + A_i^T X_{i2}^T + A_i^T Z^T - \bar{A}_f^T - C_i^T \bar{B}_f^T + \bar{P}_{i2}, \qquad \tilde{\Sigma}_{i22}^{13} = (\bar{A}_f)^\star + \bar{P}_{i3}$$

$$\tilde{\Sigma}_{i22}^{21} = A_i^T Y_{i1}^T + A_i^T Z^T - \bar{A}_f^T - C_i^T \bar{B}_f^T + P_{i1} - X_{i1}$$

$$\tilde{\Sigma}_{i22}^{22} = A_i^T Y_{i2}^T + A_i^T Z^T - \bar{A}_f^T - C_i^T \bar{B}_f^T + P_{i2} - Z$$

$$\tilde{\Sigma}_{i22}^{23} = \bar{A}_f^T + P_{i2}^T - X_{i2}, \qquad \hat{\Sigma}_{i22}^{24} = \bar{A}_f^T + P_{i3} - Z$$

the others are given in Theorem 3*. Then, the parameters of filter* (35) *can be constructed by*

$$A_f = Z^{-1}\bar{A}_f, \qquad B_f = Z^{-1}\bar{B}_f, \qquad L_f = \bar{L}_f \tag{38}$$

Table 1
Allowable upper bounds of $\overline{\mu}$ for different α with given $\underline{\mu} = 0.1$.

α	0	0.4	0.7	0.8	1
$\overline{\mu}$	1.65	1.68	1.7	1.73	1.81

5. Numerical examples

In this section, an example is used to demonstrate the applicability of the proposed approach.

Example 1. Consider a stochastic Markovian jump system with mode-dependent time-varying delays of form (1) with tow modes. For mode 1, the dynamics of system are described by

$$A_1 = \begin{bmatrix} -3 & 1 & 0 \\ 0.3 & -4.5 & 1 \\ -0.1 & 0.3 & -3.8 \end{bmatrix}, \quad A_{d1} = \begin{bmatrix} -0.2 & 0.1 & 0.6 \\ 0.5 & -1 & -0.8 \\ 0 & 1 & -2.5 \end{bmatrix}, \quad B_1 = \begin{bmatrix} 1 \\ 0 \\ 1 \end{bmatrix}$$
$$G_1 = \begin{bmatrix} 0.1 & -0.1 & 0.2 \\ 0.3 & 0.3 & -0.4 \\ 0.1 & 0.1 & -0.3 \end{bmatrix}, \quad G_{d1} = \begin{bmatrix} 0.1 & -0.1 & 0.2 \\ 0.3 & 0.3 & -0.4 \\ 0.1 & 0.1 & -0.3 \end{bmatrix}$$
$$C_1 = \begin{bmatrix} 0.8 & 0.3 & 0 \end{bmatrix}, \quad C_{d1} = \begin{bmatrix} 0 & -0.6 & 0.2 \end{bmatrix}, \quad D_1 = 0.2$$
$$H_1 = \begin{bmatrix} 0.2 & -0.1 & 0.3 \end{bmatrix}, \quad H_{d1} = \begin{bmatrix} 0.2 & -0.1 & 0.3 \end{bmatrix}, \quad L_1 = \begin{bmatrix} 0.5 & -0.1 & 1 \end{bmatrix}$$

For mode 2, the dynamics of the system are described as

$$A_2 = \begin{bmatrix} -2.5 & 0.5 & -0.1 \\ 0.1 & -3.5 & 0.3 \\ -0.1 & 1 & -3.2 \end{bmatrix}, \quad A_{d2} = \begin{bmatrix} 0 & -0.3 & 0.6 \\ 0.1 & 0.5 & 0 \\ -0.6 & 1 & -0.8 \end{bmatrix}, \quad B_2 = \begin{bmatrix} -0.6 \\ 0.5 \\ 0 \end{bmatrix}$$
$$G_2 = \begin{bmatrix} 0.1 & -1 & 0.2 \\ 0.3 & 0.3 & -0.4 \\ 1 & 0.1 & 0.3 \end{bmatrix}, \quad G_{d2} = \begin{bmatrix} 0.1 & -0.1 & 0.2 \\ 0.3 & 0.3 & -0.4 \\ 0.1 & 0.1 & -0.3 \end{bmatrix}$$
$$C_2 = \begin{bmatrix} -0.5 & 0.2 & 0.3 \end{bmatrix}, \quad C_{d2} = \begin{bmatrix} 0 & -0.6 & 0.2 \end{bmatrix}, \quad D_2 = 0.5$$
$$H_2 = \begin{bmatrix} 0.2 & -0.1 & 0.3 \end{bmatrix}, \quad H_{d2} = \begin{bmatrix} 0.2 & -0.1 & 0.3 \end{bmatrix}, \quad L_2 = \begin{bmatrix} 0.5 & -0.1 & 1 \end{bmatrix}$$

The transition rate matrix is given as

$$\Pi = \begin{bmatrix} -0.5 & 0.5 \\ 0.3 & -0.3 \end{bmatrix}$$

In [22–25], the obtained filter is totally mode-dependent, which must need the system mode obtained exactly online and limits the scope of application of the obtained results. However, by Theorem 3, we can design a partially mode-dependent filter instead, where the system mode is unnecessary to filter implementation. It means that the desired filter can suffer system mode lost with some probability, even if the system mode is always unavailable. With given $\gamma = 1.2$ and mode-dependent time-varying delay $\tau_{\eta_t}(t)$ satisfying (4) with $h_1 = 0.2$, $h_2 = 0.3$, Table 1 presents the comparison result for different α with $\underline{\mu} = 0.1$. It is seen that the larger value of α means that the larger uppermost bound of delay $\overline{\mu}$, which predicates the corresponding criterion has less conservatism. That is because the stochastic property of mode signal available to filter is considered in filter design, which bridges the two extremely traditional filtering methods: totally mode-dependent and mode-independent methods. In addition, a potential application of the proposed method is used to reduce the burden of data transmission in many practical systems, such as NCSs, where the data are transmitted through unreliable networks and suffers packet loss. When the corresponding system is an MJS and the system mode is also transmitted through networks, the mode-dependent filtering approach is too ideal and is no longer applied to the above case. However, the presented partially mode-dependent filtering method is very suitable to deal with this case, where the mode signal suffers lost or discarded forwardly in terms of some distribution probability to attain more expedite of networks. Especially, even if the system mode is always inaccessible, via Corollary 1, a totally mode-independent filter without mode information can also be determined via mode-dependent Lyapunov function, whose parameters are

$$A_f = \begin{bmatrix} -2.4886 & -3.3131 & 3.9652 \\ 0.5317 & -2.4921 & -0.3368 \\ 1.3018 & 4.0401 & -6.6568 \end{bmatrix}, \quad B_f = \begin{bmatrix} -1.6736 \\ 1.8906 \\ 5.2643 \end{bmatrix}, \quad L_f = \begin{bmatrix} 0.2142 \\ 0.2084 \\ 0.3607 \end{bmatrix}^T$$

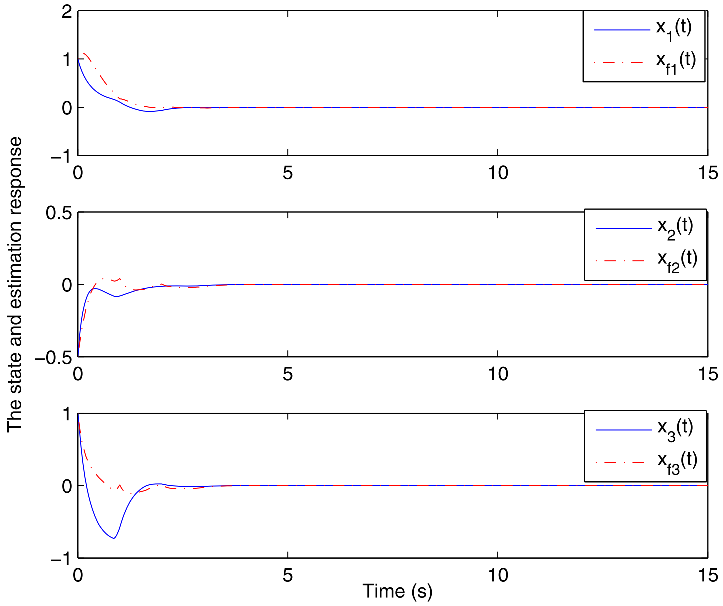

Fig. 1. The responses of state $x(t)$ and estimation $x_f(t)$.

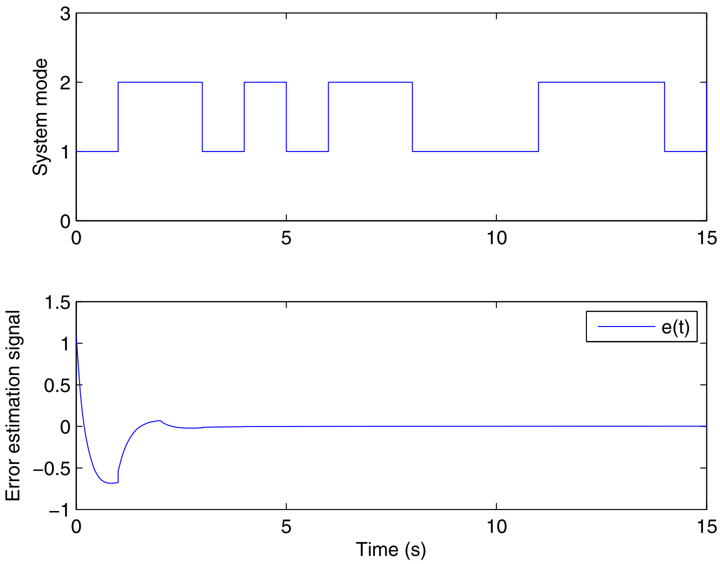

Fig. 2. The system mode η_t and error estimation signal $e(t)$.

Let initial condition $\hat{x}_0 = [\,1\ {-0.5}\ 1\ 1\ {-0.5}\ 1\,]^T$ and applying the achieved filter, the responses of system state and estimation are given in Fig. 1. In addition, the simulations of system mode and error estimation signal are shown in Fig. 2, which demonstrates the desired H_∞ filter approximates the original model very well.

6. Conclusions

In this paper, we have investigated the H_∞ filtering problem for a class of continuous-time stochastic Markovian jump systems with mode-dependent time-varying delays. Instead of using mode-independent Lyapunov function method or assuming Lyapunov matrix with special form, a new kind of H_∞ filter named as partially mode-dependent filter is established, which bridges the following two extreme cases, mode-dependent and mode-independent filter design methods. Sufficient criteria for H_∞ filter are given in terms of LMIs. Finally, numerical example is prosed to illustrated the utility and advantage of the developed theories.

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